ABSTRACT: A combinatorial optimization procedure for best management practice (BMP) placement at the watershed level facilitates selection of cost effective BMP scenarios to control nonpoint source (NPS) pollution. A genetic algorithm (GA) was selected from among several optimization heuristics. The GA combines an optimization component written in the C++ language with spatially variable NPS pollution prediction and economic analysis components written within the ArcView geographic information system. The procedure is modular in design, allowing for component modifications while maintaining the basic conceptual framework. An objective function was developed to lexicographically optimize pollution reduction followed by cost increase. Scenario cost effectiveness is then calculated for scenario comparisons. The NPS pollutant fitness score allows for evaluation of multiple pollutants, based on prioritization of each pollutant. The economic component considers farm level public and private costs, cost distribution, and land area requirements. Development of a sediment transport function, used with the Universal Soil Loss Equation, allows the optimization procedure to run within a reasonable timeframe. The procedure identifies multiple near optimal solutions, providing an indication of which fields have a more critical impact on overall cost effectiveness and flexibility in the final solution selected for implementation. The procedure was demonstrated for a 1,014-ha watershed in the Ridge and Valley physiographic region of Virginia.

KEY TERMS: watershed management; genetic algorithm; spatial optimization; modeling; geographic information systems; nonpoint source pollution; sediment delivery.


INTRODUCTION

In the last few decades there has been increasing concern over water and soil borne pollutants that influence human or aquatic health or that restrict human activities. In particular, nonpoint source (NPS) pollution from agricultural lands contributes significantly to water quality degradation. Government regulations, such as the Clean Water Act, are placing growing emphasis on NPS pollution control. One method of control is through implementation of best management practices (BMPs) – structural, vegetative, or cultural methods by which NPS pollution is eliminated or reduced sufficiently to meet water quality criteria (Novotny and Olem, 1994).

Implementation of all applicable BMPs on each field within a watershed would help control NPS pollution to the greatest extent possible with the current level of scientific knowledge. However, extensive, widespread BMP applications can be cost prohibitive and redundant in pollution control. Best management practice implementation often increases both farmer and public costs. For example, many BMPs require using different machinery, building structures, and learning new techniques, and might also reduce yield. Additionally, BMPs might be subject to governmental contracting and inspection to meet legal or cost share requirements.

Success in locating the most cost effective BMP scenario for a specific watershed depends on the ability to consider the complete range of possible scenarios within a watershed, accounting for spatial variation, field specific BMP effectiveness, and BMP interaction within and among fields. However, the number of ways to allocate BMPs throughout a watershed is exponential with regard to the number of fields. For example, for 50 fields and four nonmutually exclusive BMPs, there are \(24^{50}\) possible placement scenarios. Evaluation of all possible BMP scenarios becomes an
intractable problem, one that is computationally difficult or impossible to solve for an exact solution in a finite amount of time.

Evaluating and comparing a small number of potential BMP scenarios through expert judgment, even with the aid of a geographic information system (GIS) and analysis software, are time consuming and subject to judgment inconsistencies. Advances in computational speed and software now enable evaluation of a large sample of possible scenarios for a watershed in a reasonable timeframe. Using an optimization heuristic to determine scenario effectiveness eliminates the laborious task of individual evaluation and lessens computational errors and evaluation inconsistencies. In particular, Srivastava et al. (2002) have shown the potential of the genetic algorithm (GA) in locating scenarios that reduce pollution or farmer costs as compared to multiple random scenarios. Additionally, because the optimization procedure evaluates the complete BMP scenario, each scenario can theoretically be limited to precisely the combination of BMPs necessary to meet the water quality criteria.

The overall goal of the paper is to describe development of an optimization procedure that identifies BMP combinations at a watershed scale that meet specified pollutant reduction levels while minimizing costs. The specific objective of this research was to create an optimization procedure that would: (1) place sufficient BMPs on the watershed to meet water quality criteria; (2) limit each scenario, as much as possible, to the combination of BMPs necessary to meet the water quality criteria; and (3) identify the lowest cost scenario possible.

PROCEDURE DEVELOPMENT

Heuristic Selection

Existing optimization heuristics for solving intractable problems include gradient and nongradient based neighborhood searches as well as methods developed from studies of natural systems. To determine a basic heuristic well suited to this problem, five heuristics for solving intractable problems were considered: GA, response surface methodology, shuffled complex evolution, simulated annealing (SA), and tabu search. Several factors were compared among the heuristics, including performance for similar types of problems in previous studies, proof of convergence, and ease of formulation. Next, each heuristic's continuity and differentiability requirements, convergence rate, and relative efficiency were considered, as were sensitivity of the heuristic to the problem formulation and the number of points needed as a starting requirement. In addition to the above heuristics, the use of a classical method, such as integer programming or nonlinear optimization (as used by Braden et al., 1989), was considered briefly.

Overall, the problem was determined to be most simply suited to characterization as a combinatorial optimization problem (Lawler, 1976; Grötschel, 1982). Compared to the other techniques considered, the GA and SA seemed more straightforward to formulate in a manner that could accommodate evaluation of different watersheds. Because the GA and SA do not require continuity or differentiability, they are well suited to the combinatorial aspect of this problem. Both the GA and SA have been proven to converge arbitrarily close to the optimum under certain assumptions (Lundy and Mees, 1986; Siegelmann and Frieder, 1991). Additionally, unlike the tabu search, the GA and SA do not require problem specific selection rules.

At each generation, the GA evaluates multiple scenarios, often from different areas of the search space. This parallelism decreases susceptibility to becoming fixed at local minima (Buckles and Petry, 1992). Additionally, by looking at the most fit scenarios in a given generation, a policy maker can determine which fields tend to be managed in the same way across scenarios and which fields vary in management across scenarios. Since the most fit scenarios are selected from across the search space, it is likely that the fields that are managed the same across scenarios have a greater impact on total watershed loadings than the fields for which management varies across scenarios.

Additionally, subsequent evaluations, drawing from scenarios across the breadth of the search space, help identify fields within the watershed that appear to have a greater impact on the total watershed quality and those that can vary in management practice with less overall impact.

Convergence rate and relative efficiency of the GA, in comparison with the other heuristics, were not clear. Performance of the GA in these two areas appeared no better or worse than that of the majority of the other heuristics and to, perhaps, be dependent on the specific problem and/or problem formulation. The GA, like the other heuristics, was seen to be sensitive to problem formulation. Previous work with the GA in placement of management practices (Srivastava et al., 2002) was available to provide some insight into a possible problem formulation. Based on a subjective comparison of the heuristics with regard to the cited factors (Veith, 2002), the GA was selected for use in the optimization procedure.

The GA is conceptually based on natural selection techniques seen in biological evolution (Goldberg,
scenarios were formulated as two-dimensional binary strings where a binary string for each field identified the field’s rotation. Fifteen cropland rotations were considered. In the formulation described here, use of allele sets in each scenario representation allows increased specificity of management practice options for each land use type. That is, management practices appropriate only to cropland, or even to a specific type of crop, can be excluded from consideration on hayland and other areas.

**Problem Formulation**

To formulate the BMP location problem for GA optimization, each watershed scenario can be thought of as a chromosome. A possible solution to the problem is represented as a chromosome and each land use area, including crop/cover and BMPs, is represented as a gene on that chromosome. The value of each gene, representing the land cover and management practice or combination of practices, along the chromosome is chosen from a set of possible values, or alleles, for that gene.

In the watershed scenario representation, the chromosome is written as an array of numbers. Each position of the array holds the identification value for a corresponding field or management unit. The value in that position represents the specific management practice on that field, while the list of all acceptable management practices for that field forms the allele set for that array position.

Each scenario, evaluated for pollution reduction and cost increase, is assigned an ordered value of cost effectiveness. The BMP location problem can then be formulated as an unconstrained optimization problem with an objective function that maximizes cost effectiveness.

The GA is initialized with a random population of scenarios where each scenario is subject to the constraints of the allele array. Thus, any land use area not in production is assigned a single allele and maintains a fixed set of management practices. Land use areas in production are assigned a set of alleles, corresponding to the set of acceptable BMPs.

The baseline scenario is the scenario to which each new scenario is compared. Baseline management practices can come from the current land uses and management practices in the watershed, from the profit maximizing scenario (most profitable management practice for each field), or from any other scenario of choice.

In Srivastava et al.’s (2002) problem formulation, scenarios were formulated as two-dimensional binary
Through ArcView scripts, the NPS and economic components determine the pollutant load at the outlet and the scenario cost. This information is then passed back to the DLL where the scenario fitness is assigned. When the GA has met the termination criterion, control is returned to the main ArcView script.

**Optimization Component**

The optimization component translates the results of the NPS prediction component into a pollutant fitness score for each scenario based on pollution reduction. The optimization component also translates the economic analysis component results into an economic fitness score. The combined fitness score for each scenario is compared with the fitness scores of other scenarios.

**Pollution Reduction.** In the interest of assigning BMPs to decrease NPS pollution from a baseline, pollution increase at the watershed outlet, as a result of altering a BMP assignment, was not an acceptable option. Such a scenario would be no better than the baseline scenario in terms of pollution. Thus, a pollution fitness score was developed that focused on positive pollution reduction. The pollution fitness score scales the pollutant loading of each scenario relative to the loading goal. As a result, all scenarios that increase pollution as compared to the baseline are given a pollution fitness score of zero. Scenarios that reduce pollution are given a positive fitness score. The fitness score for a single pollutant is calculated by

\[
P_i = \begin{cases} 
0 & \text{for } z_b \leq z_w \\
\frac{z_b - z_w}{z_b - z_t} & \text{for } z_t < z_w < z_b \\
1 & \text{for } z_w \leq z_t 
\end{cases} \tag{1}
\]

where \( p_i \) = fitness score of pollutant \( i \), \( z_b \) = pollutant loading from baseline scenario (Mg), \( z_w \) = pollutant loading from working scenario (Mg), and \( z_t \) = maximum pollutant loading goal (Mg).

In the case of multiple pollutants, a unique pollutant loading criterion can be set for each pollutant and the individual pollutant fitness scores weighted relative to each other in terms of importance

\[
P = \frac{\sum \beta_i p_i}{\sum \beta_i} \tag{2}
\]

where \( P \) = total pollutant fitness score, \( \beta_i \) = weighting factor of pollutant \( i \), \( p_i \) = fitness score of pollutant \( i \), and \( \sum \beta_i = 1 \). By incorporating pollutant reduction goals as inputs, the optimization procedure maintains flexibility to numbers and types of pollutants, pollutant weightings, and reduction goals.

**Cost Increase.** While it was anticipated that costs would increase from the baseline as BMPs were added, a scenario meeting the pollution reduction criterion and decreasing cost would certainly be acceptable. Thus, in modeling the scenario cost, the economic fitness score had to allow for both increase and decrease in cost as compared to the baseline. Economic fitness score calculations are relative to opportunity costs (i.e., the costs of not adopting the management practice with the highest net return). Thus, a scenario’s economic fitness score remains positive even if the cost decreases below the baseline scenario. However, cost increase, expressed as change in cost relative to the baseline, may be positive or negative.

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**Figure 1. Flow Chart Showing Linkage of the Three Optimization Procedure Components.**

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Agricultural BMPs can affect the crops and forages produced by farms in a watershed. Thus, it was important that solutions not be chosen based on pollution reduction and cost alone, but that they also conform to reasonable farming practices. To facilitate this, two additional criteria were addressed in formulating the objective function.

First, preference was given towards farms meeting feed production and nutrient management requirements. Implementation of a farm area requirement ensures that the optimization program assigns fields into cropland and hay/pasture as necessary for the farm to produce sufficient amounts of feed and have sufficient grassland available for manure/litter spreading. Allocation amounts vary based on farm type (such as beef, dairy, or poultry).

Second, farm level cost fairness was incorporated to prevent, as much as possible, a solution from placing extreme cost burdens on a few farmers. The optimization procedure minimizes cost increase per pollution reduction at the watershed scale. Spurlock and Clifton (1982) demonstrated that an NPS pollution control strategy based on cost increase per unit of pollution reduction is economically more equitable to farmers than a strategy based on pollution reduction per unit area. To incorporate this information, the economic fitness score was developed such that for scenarios of near equivalent cost increase, the scenario involving the most farmers and distributing costs most equally across the affected farmers is preferred.

The economic fitness score was developed specifically for this optimization procedure to address the identified issues and criteria. It was structured to consider public and private costs as well as farm level area requirements and cost fairness, and is expressed as

$$E = 1 + \frac{C_o}{\sqrt{\sum_i \frac{x_i^2}{a_i} + 1}}$$

where $E =$ economic fitness score; $C_o =$ total opportunity cost for all farms in scenario($) ; $x_i =$ cost of working scenario for farm $i$ ($) ; $a_i =$ area required for farm $i$ (ha), and $ab =$ extent to which baseline scenario meets area requirements of farm $i$.

$$aw = \left( \frac{1}{n} \sum_r \min \left( \frac{a_{ow}}{a_e} , 1 \right) \right)_{r=1}^{n} \text{ for } n > 0, r = 1 \text{ to } n$$

$$ab = \left( \frac{1}{n} \sum_r \min \left( \frac{a_{ob}}{a_e} , 1 \right) \right)_{r=1}^{n} \text{ for } n > 0, r = 1 \text{ to } n$$

where $a_{ow} =$ area in working scenario contributing toward requirement $r$ for farm $i$ (ha), $a_e =$ area required under requirement $r$ for farm $i$ (ha), and $ab =$ extent to which baseline scenario meets area requirements of farm $i$.

The opportunity cost of the baseline scenario is used to scale the function to allow for different ranges of costs in each optimization run.

The Euclidean distance metric was used in the economic fitness function to help distribute the impact of cost increase among farms. Using this metric instead of simply adding private costs across farms results in a more preferable score when several farms each incur a little cost than when a single farm incurs the equivalent cost. However, since each farm incurs the public cost once for one or more BMPs adopted, less total public cost is incurred if the management practice changes are distributed over as few farms as possible. The result is, roughly, that the economic fitness function tends to prefer change in a limited number of farms while preferring cost increases to be distributed as equally as possible among those farms.

A farm may or may not meet area requirements in the baseline or in a working scenario, depending on the farm type and size and on the management of production land. For example, a dairy farm that requires all production land to be in corn to supply sufficient feed will no longer meet the area requirement if one of the fields is changed to hay in the working scenario. When the area requirement percentage met by a farm in the working scenario is less than that met by the baseline, the farm’s contribution to the economic fitness function increases, resulting in a decrease in the economic fitness function.

**Cost Effectiveness.** The cost effectiveness ratio – pollution reduction over cost increase – can be written as $\frac{p}{c}$ where $p$ and $c$ are real numbers. Mathematically, however, $(\frac{p}{c})'(c) = \frac{p}{c}$, where $p$ and $c$ are positive real numbers. This relationship implies, for example, that reducing 10 Mg/ha of sediment for a cost increase...
of $100 is equal in cost effectiveness to increasing sediment by 10 Mg/ha but decreasing costs by $100. In the first situation, both terms are positive with regard to the cost effectiveness definition, whereas in the second situation, both terms are negative. Thus, both ratios equal 0.10. Additionally, it is implied mathematically that both situations are more cost effective than reducing 10 Mg/ha of sediment while decreasing costs by $100 (0.10 Mg/ha/$ versus -0.10 Mg/ha/$). However, the latter situation is certainly both environmentally and economically preferable. Thus, using the cost effectiveness ratio as a single objective function for the GA does not clearly define the response surface to the research problem.

As pollutant reduction criteria are introduced, the scenario preference becomes dependent on which of the scenarios, if any, meet the reduction criteria. To solve this problem, the single cost effectiveness ratio objective function was split into a two-part multi-objective problem: (1) meet or exceed the pollution reduction criterion ($P$ from Equation 2) and (2) minimize cost increase ($E$ from Equation 3). The two objectives were reconciled into a single set of objective functions using a lexicographic method (Roumasset, 1976; Rentmeesters et al., 1996; Coello, 2000). In this method, the objective functions are prioritized in some manner, such as by desirability or importance, and solved sequentially.

The objective function combines the pollutant and economic fitness scores to evaluate each scenario:

$$F = \begin{cases} P & \text{for } P < 1 \\ E & \text{for } P = 1 \end{cases}$$

(6)

where $F$ = objective function (combined fitness score). Each scenario is first examined to see if its pollutant load meets all pollutant targeting criteria. All scenarios that meet the pollutant targeting criteria have a total pollutant fitness score of one and are ranked based on their economic fitness scores. Thus, their combined fitness scores equal their economic fitness scores, which range from one to $(1 + C_o)$. All scenarios not meeting the pollutant targeting criteria are ranked by their total pollutant fitness scores so that their combined fitness scores equal their total pollutant fitness scores, which range from zero to one. Hence, for each population and for the GA as a whole the scenario that meets all pollutant targeting criteria and farm area requirements for the least cost has the highest combined fitness score.

**NPS Component**

Two criteria were established for the NPS component of the optimization procedure:

1. Sufficient within field variation should be incorporated to compare the impacts of BMPs with regard to their location in the watershed and to utilize the spatial data available.

2. Computer run time should not exceed one day for small watersheds with few (less than 10) management alternatives per field when using a 1.6 Ghz computer.

To meet the first criterion, each watershed was discretized into cells smaller than most fields (0.09-ha cells). Current NPS models (e.g., ANSWERS-2000; Bouraoui and Dillaha, 1996), with adequate levels of discretization require prohibitive amounts of computer run time for the number of evaluations needed by an optimization heuristic. Thus, to meet the second criterion under this level of discretization, an NPS component was developed to determine cell level gross erosion and route eroded sediment to the watershed outlet through downstream overland and channel cells. Use of a GIS enabled the desired level of discretization and facilitated routing and simultaneous cell level calculations across the watershed.

**Gross Erosion.** Gross erosion is modeled in the GIS using the Universal Soil Loss Equation (USLE) (Schwab et al., 1993).

$$A = RKSCLP$$

(7)

where $A$ = average annual soil loss (Mg/ha), $R$ = combined rainfall and runoff erosivity $(\text{MJ} \cdot \text{mm} \cdot \text{ha}^{-1} \cdot \text{h}^{-1} \cdot \text{y}^{-1})$, $K$ = soil erodibility $(\text{Mg} \cdot \text{ha}^{-1} \cdot \text{h}^{-1} \cdot \text{MJ}^{-1} \cdot \text{ha}^{-1})$, $S$ = slope steepness factor, $L$ = slope length factor, $C$ = cover-management factor, and $P$ = supporting practices factor.

The $S$ and $L$ factors are calculated as (Schwab et al., 1993)

$$S = 10.8 \sin \Theta + 0.03 \quad \text{for } \Theta < 5.14 \text{ degrees}$$

$$S = 16.8 \sin \Theta - 0.50 \quad \text{for } \Theta \geq 5.14 \text{ degrees}$$

(8)

where $\Theta$ = slope steepness in degrees, and

$$L = \left( \frac{L}{22} \right)^{n}$$

(9)
where \( l = \) slope length (meters), and \( m = L\)-factor exponent = \( \frac{\sin \Theta}{\sin \Theta + 0.269 (\sin \Theta)^{0.8} + 0.05} \).

Required data for the gross erosion model include the USLE \( R \) and \( K \) factors, a digital elevation model (DEM), management unit boundaries, and land use and management practices for each unit. Required data for the USLE \( S \) and \( L \) factors include slope steepness, obtainable from a DEM, and characteristic field slope length, obtainable from a local resource conservationist or from field measurements. Additionally, the USLE \( C \) and \( P \) factors must be defined for each crop management practice to be considered.

**Sediment Routing.** A sediment routing component was developed to account for downstream effects on sediment delivery. To account for interactions among neighboring BMPs, spatial variation in sediment delivery was considered at the smallest available level, the GIS cell. A delivery ratio for each GIS cell was calculated and applied both to gross erosion generated within a cell and to sediment flowing into a cell. Delivery from each cell was then routed along the flow path to obtain the net sediment yield of each cell to the watershed outlet.

Separate sediment delivery equations were developed for overland flow and for two types of channels: shallow concentrated flow and stream flow through ephemeral and perennial streams. Sediment delivery through overland flow cells is modeled as a function of land use cover, slope steepness, and flow length:

\[
d = \min \left\{ \alpha \sqrt{\frac{s}{f}}, 1 \right\}
\]

where \( d = \) sediment delivery ratio through an overland cell, \( \alpha = \) land use coefficient (dimensionless), \( s = \) slope steepness across cell (m/m), and \( f = \) length of flow path across cell (m).

Equation (10) was developed as follows. Because the USLE predicts average annual gross erosion, consideration of nonstorm specific delivery factors was appropriate. Land cover and slope are key, nonstorm specific factors affecting delivery rates (Novotny and Olem, 1994). Thus, sediment delivery was related to overland flow velocity by modification of the flow velocity equation (Haan et al., 1994)

\[
v = \alpha s^{1/2}
\]

where \( v = \) velocity (m/s), \( s = \) slope (m/m), and \( \alpha = \) land use coefficient. This equation is nonstorm specific and applicable to overland and shallow channel flow. Also, it considers the effects of land use and slope.

Watershed level sediment delivery is a complex function of individual watershed characteristics. In particular, multiple studies, summarized by Walling (1983), have shown sediment yield at the watershed outlet to decrease as watershed area increases. Additionally, Walling (1983) summarized sediment delivery prediction equations developed for several regions of the United States. These equations proposed that sediment delivery ratios at the watershed level also decrease as watershed area increases. The prediction equations are functions of watershed area, relief, length, and slope.

The research summarized by Walling (1983) indicates that both slope and flow length are significant factors in predicting sediment delivery. Additionally, the inverse relationship between sediment delivery and watershed area suggests an inverse relationship between sediment delivery and overland flow length. Thus, to create a cell level delivery function, the right side of Equation (11) was divided by the square root of the flow length on a per cell basis. Next, a new land use coefficient, \( \alpha \), appropriate for determining sediment delivery rates, was developed to replace the land use coefficient, \( a \), from Equation (11), which is appropriate for determining velocity. The resulting equation was used to calculate cell level delivery ratios:

\[
d = \alpha \sqrt{\frac{s}{f}}
\]

where \( d, \alpha, s, \) and \( f \) are as described in Equation (10). The slope steepness and length of flow path across each overland cell are determined by a GIS.

As an empirical coefficient, \( \alpha \) can be determined using two approaches. One approach includes use of measured sediment yield or delivery data along with slope and length. Another approach is to predict sediment delivery using an NPS model. After collecting data with either method, Equation (12) can then be solved for \( \alpha \).

Data available in the literature were insufficient to determine \( \alpha \) values. Thus, the field scale NPS model, RUSLE2 (University of Tennessee, 2001) was used to estimate a delivery ratio for a slope/soil combination. First, a two-section slope profile (Figure 2) was modeled in RUSLE2. The soil of the lower section was defined as noneroding so that no gross erosion was simulated for the lower section. Erosion leaving the upper section underwent deposition in the lower section. The amount of deposition was a function of the management practice, slope steepness, and slope.
length of the lower section. This allowed the delivery ratio for the lower section to be estimated as the ratio of the net soil loss from the slope profile to the net soil loss from the upper section. Then Equation (12) was solved for $\alpha$, resulting in the values presented in Table 1.

Higher delivery is expected by channel than by overland flow due to increased flow depth, velocity, and carrying capacity. Channel cells can be identified from a DEM in terms of the number of upstream cells accumulating to create a channel cell. The use of DEMs and a flow accumulation threshold to represent the stream network is widely used in GIS applications (Garbrecht and Martz, 2000). For example, in the Ridge and Valley physiographic region of Virginia, shallow concentrated flow was identified as flow accumulated from at least 60 cells but less than 200 cells using a 30-m DEM and stream flow as flow accumulated from at least 200 cells (Veith, 2002). The entire cell containing a stream is assigned the relevant stream delivery value. Overland sediment moving to the channel is not treated separately for cells containing streams. Delivery ratios of 0.98 and 0.9998 were assigned for shallow concentrated flow and stream flow cells, respectively. These ratios were selected to represent the low level of deposition expected in small headwater, rural watersheds (T. A. Dillaha, personal communication, Biological Systems Engineering Department, Virginia Tech, Blacksburg, Virginia, March 8, 2002). For different sized watersheds it may be desirable to adjust the cell sizes or to modify the channel definitions or delivery levels.

The sediment yield contribution of each cell is determined by routing sediment from the cell through downstream cells to the outlet. For each cell, the gross erosion is multiplied by the delivery ratios of cells in the flow path from the cell to the outlet

$$Y_i = A_i a_i \prod d_j$$  \hspace{1cm} (13)

where $Y_i$ = sediment loss of cell $i$ reaching the outlet (Mg), $A_i$ = gross erosion from cell $i$ (Mg/ha), $a_i$ = area of cell $i$ (ha), $d_j$ = sediment delivery ratio of cell $j$, and $j$ indexes all flow path cells between cell $i$ and the outlet.

ArcView GIS (ESRI, 1999) does not currently provide a function for multiplying cell values along a flow path. However, the ArcView FlowLength function can be used to closely approximate Equation (13) by rewriting the product of the delivery ratios as an additive exponential function

$$\prod d_j = e^{\sum \ln(d_j)} = e^{-\text{FlowLength} \left( t_j \times \frac{-\ln(d_j)}{f_j} \right) \hspace{1cm} (14)}$$

where $d_j$ = sediment delivery ratio of cell $j$, $f_j$ = flow length assigned to cell $j$, and $t_j$ = travel distance of flow between cell $j$ and the next cell in the flow path.

The routing process is illustrated for a single cell in Figure 3. The arrows show the flow path from Cell 1, through Cells 2 and 5 to the outlet (Cell 9). Using Equation (14), the sediment delivery to the watershed outlet for Cell 1 is calculated as

$$Y_1 = A_1 a_1 d_1 d_2 d_3 d_9$$  \hspace{1cm} (15)

Summing the sediment loss reaching the outlet (i.e., the $Y_i$’s) over all cells and dividing by the watershed area gives the sediment yield of the watershed in Mg/ha. This method is similar to that used by Kothyari and Jain (1997) for routing sediment in forested watersheds.

**Sensitivity to Spatial Placement of Management Practices.** The effect of land use placement within a watershed on sediment yield was determined to conform to expected trends in the routing portion of the NPS component. It was expected that sediment yield at the watershed outlet would increase when
erodible land uses were located nearer to streams or nearer to the watershed outlet. To assess spatial sensitivity, a land use layer consisting of seven agricultural fields and two larger land use regions was created (Figure 4). Each agricultural field was 3.6 ha in size. A constant USLE $K$ factor of 0.042 Mg·ha·h/(ha·MJ·mm) was assigned to eliminate variability in soil erodibility. Slopes in each field ranged from two to five percent. The USLE $C$ factor was set at 0.003 for forest, 0.01 for grass hay, and 0.49 for conventionally tilled corn silage areas. For all regions, a USLE $P$ factor of one was used. The $\alpha$ value was set at 1.1 for forest, 3.3 for grass hay, and 9.7 for corn silage.
The NPS component was used to calculate sediment loading to the outlet for one reference and seven test runs. The upper region remained in forest for all test runs. For the reference run, the lower region of the watershed, including all seven agricultural fields, was placed in grass hay. For each of the test runs, a different agricultural field was placed in conventionally tilled corn silage. The remainder of the lower region was placed in grass hay.

Differences in gross erosion among test runs (Table 2) were due to slope steepness and flow length characteristics of the cells in each field. These two factors contributed to variations in the $S$ and $L$ factors of the USLE, while all other factors of the USLE were controlled. As expected, differences in sediment yield did not vary consistently with differences in gross erosion in the watershed (Table 2). For example, for Field 1, gross erosion increased about 11 percent relative to the reference run, while sediment yield increased about 6 percent. In contrast, for Field 2, gross erosion increased about 11 percent, while sediment yield increased 18 percent, three times more than for Field 1.

Increases in sediment yield relative to the reference run were as expected based on the placement of the fields and distribution of flow along the field edge. Also, relative differences in sediment yield due to differences in field locations within the watershed followed expected trends. For example, Field 1 was located just downstream of Field 7. Both fields were bordered by the same stream and had one to two cell widths of hay buffer along most of the stream edge. As expected, watershed sediment yield when the downstream field was in corn was greater than when the upstream field was in corn (0.094 Mg/ha versus 0.089 Mg/ha).

Economic Component

The economic impact of a given watershed scenario consists of the sum of private costs, which reflect the farmers’ compliance costs due to changing management practices, and public transaction costs, which are incurred by the government in ensuring that water quality goals are being met (Carpentier et al., 1998). Private costs, incurred by each farmer as a result of applying a management practice, are first determined at the field level as opportunity cost minus net return. Opportunity cost refers to the cost of not choosing the management practice with the highest net return. The private cost for each farm is the sum of field costs for all fields in the farm:

$$c_i = \sum_j \left( o_{ij} - \left[ \sum_k \left( \sum_l y_{ijkl} s_{ijkl} - e_{ij} \right) a_{ij} \right] \right)$$

(16)

where $c_i =$ private cost for farm $i$ ($\), $o_{ij} =$ opportunity cost for farm $i$ and field $j$ ($\), $y_{ijkl} =$ yield for farm $i$, field $j$, crop $k$, and soil $l$ (qty/ha), $s_{ijkl} =$ selling price of crop $k$ on farm $i$ and field $j$ ($/qty), $e_{ij} =$ enterprise production cost for farm $i$ and field $j$ ($/ha), and $a_{ij} =$ field area for farm $i$ and field $j$ (ha).

Public costs, the sum of contracting and enforcement costs for a given scenario, are calculated for each farm for which a BMP has been added to one or more fields. Contracting costs are incurred by government agencies while forming agreements with those farmers who are required to change management practices. Enforcement costs include expenses incurred by government agencies while ensuring contract agreements are met.

<table>
<thead>
<tr>
<th>Test Run</th>
<th>Field in Corn Silage</th>
<th>Gross Erosion in Watershed (Mg/ha)</th>
<th>Gross Erosion Within Field (Mg/ha)</th>
<th>Increase in Watershed Gross Erosion Compared With Reference Run (percent)</th>
<th>Sediment Yield at Watershed Outlet (Mg/ha)</th>
<th>Increase in Sediment Yield Compared With Reference Run (percent)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ref.</td>
<td>None</td>
<td>1.400</td>
<td>N/A</td>
<td>N/A</td>
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Two additional types of public costs were considered when developing the optimization procedure: cost share and information. In cost share programs, the farmer implements an appropriate BMP and is reimbursed, in part, by a government incentive. Considered at a farm level, the impact of cost share programs cancels out. That is, the total costs per farm equal the private costs (from which the cost share amount is subtracted) plus the public costs (to which the cost share amount is added). Hence, the optimization program does not explicitly consider cost share programs.

Information costs represent the costs involved in generating the optimal solution from the baseline scenario through development and use of the optimization procedure. Since, by this definition, information costs do not vary by run of the optimization scenario, they were not considered within the optimization procedure.

**PROCEDURE DEMONSTRATION**

Performance of the optimization procedure was demonstrated on a 1,104-ha watershed in the Ridge and Valley physiographic region of Virginia (Veith, 2002). Agricultural production in the watershed is distributed across 775 ha and 18 farms: two large cattle farms, one dairy and one beef (150 cows each); six medium dairy (100 cows), one with poultry; one medium beef with poultry (70 cows); four small dairy (60 cows); five small beef (40 cows), one with poultry. Farms were divided into 125 fields of which 51 percent was in cropland, 37 percent was in hay, and 12 percent was in pasture. The remaining 239 ha of the watershed are forested (19 percent) or for residential use (4 percent).

Three optimization runs were performed to assess the procedure’s response under varying BMP placement strategies. The maximum acceptable pollutant (sediment) load was the same for all runs. However, the runs included variation in allowable choices and combinations of cropland management (conventional or minimum tillage, with or against the contour, and with or without cover crop) and forage (pasture or grass hay).

Results across the runs were similar, relative to different conditions in each run. Thus, results for a single run demonstrate a typical progression of fitness score, sediment yield, and watershed cost values during the run (Figure 5). In this run, sediment yield decreased steadily from the baseline loading until generation 140, when the maximum acceptable pollutant load of 0.64 Mg/ha was achieved. This corresponds to a pollution fitness score of 1.0. During this

![Figure 5. Comparison of Cost and Pollution Variables With Fitness Scores for a Single Optimization Run on a 1,014 ha Watershed in the Ridge and Valley Physiographic Region of Virginia.](image)
Development of this procedure revealed that representing cost effectiveness as a ratio in a single objective function does not define a clear response surface for this problem. An effective solution involved use of a lexicographic technique to prioritize a multiobjective function. Additionally, it was found that using the USLE with a sediment transport function instead of a more detailed NPS model allows the procedure to run within a reasonable timeframe. The sediment routing routine developed for the NPS component was found to respond as expected to spatial changes in land management.

The optimization procedure was demonstrated successfully on a 1,014-ha watershed. Increase in fitness corresponded with a sediment yield decrease to the maximum acceptable pollutant load, followed by decreased costs.

LITERATURE CITED


ESRI (Environmental Systems Research Institute), 1999. ArcView GIS Ver 3.2. Environmental Systems Research Institute, Redlands, California.


SUMMARY AND CONCLUSIONS

A functional procedure was developed to optimize BMP placement based on cost and NPS pollution reduction for a watershed. This provides a computerized method for locating scenarios for which alternative BMP placements increase watershed level cost effectiveness.

Among a range of optimization heuristics, the GA and SA heuristics have features most suited to this problem type. Because the GA provides multiple solutions that meet the objectives, there is flexibility in selection of the most suitable solution based on the priorities of farmers and other stakeholders. Additionally, comparison of the final solutions can lead to an indication of those fields that are more critical to the overall watershed cost effectiveness.